

W -boson unitarity effects and oscillations in the neutral Kaon system

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Abstract

We study the effects on oscillations in the neutral-Kaon system from lepton and hadron self-energy loops in the W -boson propagators. The corresponding W box diagrams are evaluated by attaching the external quark lines to covariant vertex functions for the composite Kaons, integrating over all off-shell momenta. We find that the ratio of the imaginary and real parts of the amplitude is of the same size as the experimental value of the modulus of the CP -violating parameter ε .

The theory of weak decays, initially formulated by Fermi [1, 2] in terms of a four-fermion vertex for the description of neutron β decay, is nowadays understood by a renormalizable [3, 4] non-abelian gauge theory [5, 6, 7], in which the weak interactions are mediated by the heavy gauge bosons. Here we shall concentrate on K^0 - \bar{K}^0 oscillations via double W exchanges.

The phenomenon of a long-lived component in the two-pion decay mode of the neutral Kaon system, discovered by Christenson, Cronin, Fitch, and Turlay [8], is in the Standard Model (SM) parametrized by the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) [9, 10] matrix, which is related to the complex ε parameter. This ε measures the fraction of pion pairs observed

in the decay products of the long-lived neutral Kaons [11, 12]. Many authors have studied this phenomenon, see e.g. Refs. [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29], just to mention a few. But its dynamics remains to be discovered.

The masses of the heavy gauge bosons, which can be estimated from Fermi's weak coupling constant, are experimentally determined with great precision [30], ever since their first discovery in 1983 by the UA2 collaboration [31]. Furthermore, the branching ratios of the W s and Z are well known [30] for most of the decay modes that could play a role at Kaon energies.

In scattering theory, unstable particles are described by Breit-Wigner amplitudes [32], associated with complex-energy poles in the S matrix for the decay products [33]. Consequently, the physical, on-shell W boson, which besides being massive (mass M_W), is also unstable (width Γ_W), should be characterized by the complex quantity $M_W - \frac{1}{2}i\Gamma_W$.

In this paper, instead of describing the oscillations in the neutral Kaon system by a complex CKM phase and the exchange of massive W bosons with purely real mass M_W , we study whether a comparable result may be obtained by considering only real CKM matrix elements [34], but allowing the exchanged off-shell W bosons to develop a complex self-energy whenever this is kinematically demanded by four-momentum conservation and the known decay thresholds of the W . The crucial point here is that “*decay is a profoundly irreversible process*” [35], which should also be taken into account in virtual exchanges, especially when hunting after a tiny effect. In order to have a reliable off-shell calculation, we shall sandwich the standard box diagrams between covariant composite-Kaon vertex functions that are parameter-free, after being tuned to the experimental Kaon size. Note that the Kaon's compositeness has recently been suggested to even give rise to CP violation [36].

In principle, one could calculate the desired amplitude by directly determining the two-pion branching ratio of the long-lived neutral Kaon [37]. However, such a calculation would involve the unknowns of QCD at low energies, which causes the result to depend on the specific modeling of strong interactions. Here, we determine the lowest-order processes that transform neutral Kaons into neutral anti-Kaons, schematically represented by the diagrams T and S of Figs. 1 and 3, respectively, and compare our result with the modulus of ε . From experiment [38] one derives $|\varepsilon| = 0.0023 \pm 0.0002$.

The strategy to obtain the modulus of ε from a microscopic description stems from Gaillard and Lee [39]. It is based on a conjecture of Wolfenstein [40] that all CP -violating processes stem from double-strangeness-changing ($\Delta S = 2$) superweak interactions. The transition matrix

elements for neutral-Kaon oscillations are related to the modulus of ε by

$$|\varepsilon| = \frac{1}{2} \frac{\langle K^0 | \bar{K}^0 \rangle - \langle \bar{K}^0 | K^0 \rangle}{\langle K^0 | \bar{K}^0 \rangle + \langle \bar{K}^0 | K^0 \rangle} , \quad (1)$$

which equals half the ratio of the imaginary and real parts of the amplitude.

The original calculus [39] was carried out by taking the external quarks free and the external momenta zero. However, we know that free quarks do not exist. Moreover, the neutral Kaons have long lifetimes — on a hadronic scale — thus justifying a bound-state approach with respect to their wave functions from strong interactions. Finally, the intermediate state (e.g. $u\bar{u}$) may have a Kaon-like distribution w.r.t. strong interactions, but the involved u (or c, t) and \bar{u} (or \bar{c}, \bar{t}) quarks can have all possible momenta, only governed by total four-momentum conservation. This picture is to be contrasted with the on-shell $\Delta I = 1/2$ $K_S \rightarrow \pi^+ \pi^-$ decay via an intermediate σ resonance [41]. Hence, also for the intermediate state one must expect rearrangement effects in the quark-antiquark strong-interaction distributions. Therefore, we shall take the external quark momenta of the box diagram in agreement with the bound-state picture for neutral Kaons. As a consequence, our quarks will be dressed, and so massive. We take for the constituent quark masses the reasonable values $m_n = m_u = m_d = 340$ MeV and $m_s = 490$ MeV [42]. In any case, we shall see later that the results hardly depend on these specific values. The initial and final

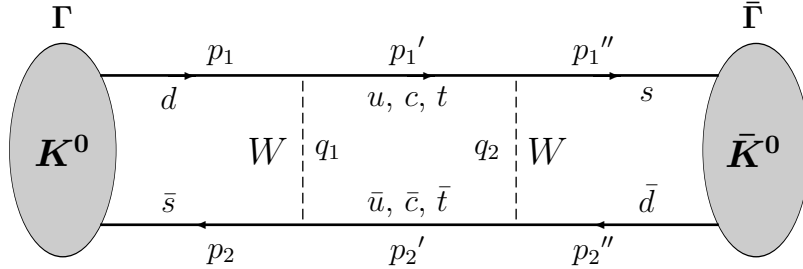


Figure 1: Diagram T : The t -channel box diagram for $K^0 - \bar{K}^0$ oscillations.

quark-antiquark bound states couple, through the vertex functions, to the quark and antiquark participating in the W -boson exchanges. The precise details of the vertex functions must be deduced from a microscopic model. We assume here that the vertex functions Γ and $\bar{\Gamma}$ depend on the relative quark-antiquark four-momenta p and p'' , respectively, and on the total center-of-mass (CM) momenta $P = p_1 + p_2$ resp. $P'' = p_1'' + p_2''$, i.e.,

$$\Gamma = \Gamma(p, P) \quad \text{and} \quad \bar{\Gamma} = \bar{\Gamma}(p'', P'') \quad . \quad (2)$$

Total four-momentum conservation is expressed by

$$P = P' = p_1' + p_2' = P'' \quad . \quad (3)$$

For the K^0 and \bar{K}^0 vertex functions Γ resp. $\bar{\Gamma}$ we assume a Euclidean Gaussian distribution ($\exp[-\frac{1}{2}\alpha p^2]$), with $\alpha = 5.37 \text{ GeV}^{-2}$, which in the nonrelativistic limit corresponds to the experimental Kaon charge radius of 2.84 GeV^{-1} [30], and is, moreover, in good agreement with the predictions of the quark-level linear σ model [43] and also with the wave functions obtained in a unitarized meson model [44].

The relations between the particle momenta p_1 and p_2 and the relative particle momenta p are given by the standard expressions

$$p_{1,2} = \pm p + \frac{m_{n,s}}{m_n + m_s} P \quad , \quad (4)$$

and similar relations for p_1', p_2' and p' , and p_1'', p_2'' and p'' . The definitions (4) have a nonrelativistic origin, but, as our calculation is fully covariant, the final result does not depend on this particular choice, which is nonetheless very convenient for numerical reasons.

For the W -propagators we write $S_W^{(i)}(q_i, M_W)$, where the exchange momenta q_1 and q_2 are defined in Fig. 1, according to $q_1 = p_1' - p_1$ and $q_2 = p_1'' - p_1'$. The two-particle propagators $G_{q\bar{q}}(p, P)$ have the usual form

$$G_{d\bar{s}}(p, P) = \frac{1}{(p_1^2 - m_n^2 + i\epsilon)(p_2^2 - m_s^2 + i\epsilon)} \quad , \quad (5)$$

and similarly for the other two. Now, we are dealing here with the propagators of quarks, which are fermions. Hence the two-particle propagators should be of the form

$$G(p_1, p_2) = \frac{1}{(\not{p}_1 - m_1 + i\epsilon)(\not{p}_2 - m_2 + i\epsilon)} \quad . \quad (6)$$

This can be arranged by multiplying the two-particle propagators of Eq. (5) by

$$(\not{p}_1 + m_1)(\not{p}_2 + m_2) \quad . \quad (7)$$

However, for the crucial off-shell kinematical effects only the pole structure is of importance, which is the same for the fermion propagators (6) and the boson propagators (5). Then, all the factors resulting from the Dirac algebra involving expressions like in Eq. (7), and also the different SM vertices, will cancel out when evaluating the ratio of the imaginary and real parts of the amplitude. Note that working with only scalar propagators allows us to use scalar vertex functions for the Kaons, and thus to model the diagrams in an essentially parameter-free way.

The total amplitude for the process represented by diagram T is given by

$$A \propto \int d^4p d^4p' d^4p'' \Gamma G_{d\bar{s}} S_W^{(1)} G_{u\bar{u}} S_W^{(2)} G_{s\bar{d}} \bar{\Gamma}. \quad (8)$$

The integrations are performed numerically, except for some angular integrations.

The pole structure of the two-particle propagators can best be studied in the CM frame (\sqrt{s} represents the total invariant mass). If we then define

$$\omega_{n,s} = \sqrt{\vec{p}^2 + m_{n,s}^2} \quad , \quad (9)$$

we obtain for the terms in the denominator of the two-particle propagator (5) the expressions

$$\begin{aligned} p_1^2 - m_n^2 + i\epsilon &= \left(\frac{m_n}{m_n + m_s} \sqrt{s} + p_0 \right)^2 - \omega_n^2 + i\epsilon \quad , \\ p_2^2 - m_s^2 + i\epsilon &= \left(\frac{m_s}{m_n + m_s} \sqrt{s} - p_0 \right)^2 - \omega_s^2 + i\epsilon \quad . \end{aligned} \quad (10)$$

We have poles in the complex p_0 -plane for the two-particle propagator (5) at

$$\begin{aligned} (1, 2) : \quad p_0 &= -\frac{m_n}{m_n + m_s} \sqrt{s} \pm (\omega_n^2 - i\epsilon) \quad \text{and} \\ (3, 4) : \quad p_0 &= \frac{m_s}{m_n + m_s} \sqrt{s} \pm (\omega_s^2 - i\epsilon) \quad , \end{aligned} \quad (11)$$

graphically represented in Fig. (2).

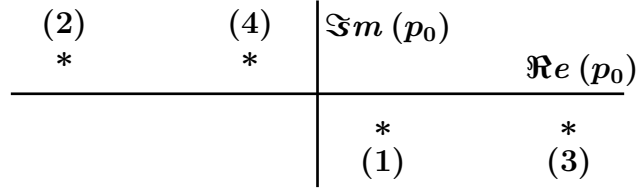


Figure 2: Singularity structure of formula (10). The poles (*) are numbered according to their appearance in Eq. (11).

Notice that $\sqrt{s} < m_n + m_s$ in the bound-state regime. Therefore, the Wick rotation $p_0 \rightarrow ip_4$, and similarly for p' and p'' , can be freely carried out (see e.g. Ref. [45]), resulting in a purely real amplitude, at least when neglecting the W self-energy effects.

For the full determination of $\langle K^0 | \bar{K}^0 \rangle$, we also include the s -channel double- W -exchange diagram depicted in Fig. 3. We obtain contributions for diagram S which in magnitude are comparable to those for diagram T . Moreover, diagram S requires less numerical effort than diagram T , since in the former case all angular integrations can be performed analytically. Consequently, checking details on how our results depend on α and the quark masses we only do for diagram S .

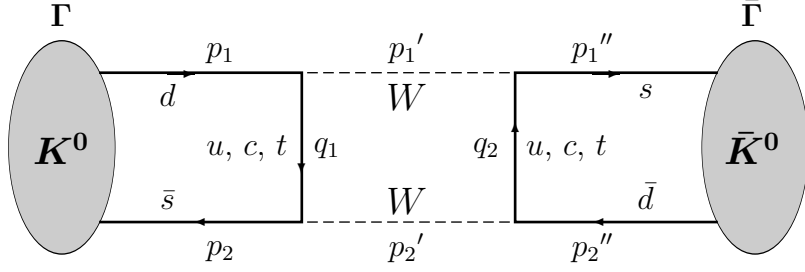


Figure 3: Diagram S : The s -channel box diagram for $K^0 - \bar{K}^0$ oscillations.

Now, let us finally come to the main issue of this paper. In order to treat the exchanged W resonances in a more realistic way than what is normally done, we first include self-energy bubbles [46] corresponding to the well-known [30] leptonic decay modes of the W , i.e., $e\nu_e$, $\mu\nu_\mu$, and $\tau\nu_\tau$, as depicted in Fig. 4.

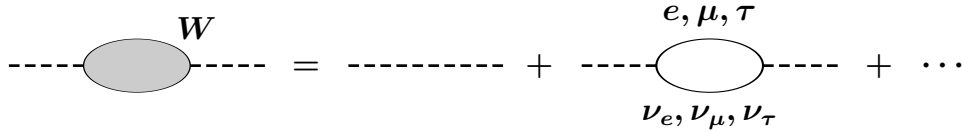


Figure 4: The full W propagator dressed with lepton loops and possibly with hadron loops

Hence, we substitute M_W^2 in the W propagators by $M_W^2 + Z_e + Z_\mu + Z_\tau$, according to

$$Z_\ell = -\frac{i}{2} \frac{\Gamma_\ell}{|p_\ell|} M_W^2 \left(1 - \frac{m_\ell^2}{p^2}\right) \Theta\left(1 - \frac{m_\ell^2}{p_{\text{Mink.}}^2}\right), \quad (12)$$

where Γ_ℓ , $|p_\ell|$, m_ℓ ($\ell = e, \mu, \tau$) and M_W are taken from Ref. [30]. Note that in the Θ function one must take the Minkowskian $p^2 = p_0^2 - \vec{p}^2$, with p_0 real, too. For obvious reasons, we have only kept the imaginary parts of the contributions to the mass self-energy (Fig. 4), the real parts being absorbed in the physical W central mass. At this point, we should also stress that adding a negative imaginary part (12) to the W mass does *not* spoil our straightforward Wick rotations, as it only amounts to substituting an infinitesimal ϵ by a finite $|Z(p^2)|$. In other words, because

of causality the sign of the imaginary part of the mass is always negative, both for stable and unstable (anti)particles [47].

In Table 1 we show how much each of the terms Z_ℓ contributes to the ratio in Eq. (1).

mode	fraction [30]	$\frac{\Im m \left(\langle K^0 \bar{K}^0 \rangle \right)}{\Re e \left(\langle K^0 \bar{K}^0 \rangle \right)}$
$e\nu_e$	$(10.72 \pm 0.16)\%$	0.00096
$\mu\nu_\mu$	$(10.57 \pm 0.22)\%$	0.00094
$\tau\nu_\tau$	$(10.74 \pm 0.27)\%$	0.00060
$c\bar{s} (D\bar{K})$	$(31^{+13}_{-11})\%$	0.00153
$cX (D\bar{B})$	$(33.6 \pm 2.7)\%$	0.00078

Table 1: Contributions to the ratio of the imaginary part over the real part of the K^0 - \bar{K}^0 amplitude from the most important W decay modes (see also text).

For the hadronic modes we make the following considerations. The only known mode containing a light hadron is $\pi\gamma$, having a decay rate of less than 0.1% [30], hence negligible. The next level of known modes contains at least one c quark. The threshold for the DK loops at 2.37 GeV is some 600 MeV higher than for $\tau\nu_\tau$, whereas the branching ratio is about the triple of the latter, which yields a contribution about two and a half times larger than from $\tau\nu_\tau$ (see Table 1).

The fraction of cX decay modes of the W is a bit larger than three times the fraction of $\tau\nu_\tau$. However, here we do not have much information on X . If we assume $X = b$, we get a contribution from DB loops which is smaller than from DK , but still considerable.

It is clear from these considerations that we cannot include the hadronic loops to a great accuracy, because of the experimental uncertainties. Nevertheless, we may conclude from our results that their total contribution will not be very different from the sum of the two individual cases included by us, which is a kind of upper bound. Nevertheless, we shall take a conservative choice for the total final theoretical error in our result. Summing the contributions of Table 1 (we actually put all loops together when determining the total $\langle K^0 | \bar{K}^0 \rangle$ amplitude), we obtain

$$\frac{\Im m \left(\langle K^0 | \bar{K}^0 \rangle \right)}{2\Re e \left(\langle K^0 | \bar{K}^0 \rangle \right)} = 0.0024^{+0.0002}_{-0.0004} \quad (13)$$

for the total contribution of W self-energy loops, which is of the same magnitude as $|\varepsilon|$ [30]. In the errors quoted in Eq. (13), we have included, besides small numerical errors, the effects of reasonable variations in the parameters α , m_n , and m_s , as well as the uncertainty in the

cX mode. Also the contributions for intermediate c and t quarks have been studied, which do not significantly alter the result (13), though adding to the total $\langle K^0 | \bar{K}^0 \rangle$ amplitude. The coincidence of the effect computed by us and $|\varepsilon|$ is very striking, the physical significance of which is open to debate.

Acknowledgments

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